Electron Rest Mass and Gravitation

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A simple expression of the relationship between the rest frame time and the proper time of a constantly accelerated point-like object can be used to express the hypothetical rest mass of any elementary or composite particle as a relativistic transformation of the electron's rest mass. This expression has its origins in Special Relativity and is used to introduce a model of small and large particle mass properties and interactions that includes gravitation. An inverse interdependence between a changing electric field and a gravitational field is suggested by the introduction of an expression that will be shown to have several constructive interpretations and applications.[1]

Values derived fall comfortably between the currently measured uncertainties when restricted to six digits of the gravitational constant.[2] *The accelerations used are very nearly constant, allowing for the use of Special Relativity in this introduction to an electro-gravitational model of particles, fields, and waves.*

Accelerated bodies in Special Relativity

Consider the simple relationship between the rest frame time *t*, and proper time τ of periodic, clock-like, phenomena that occur within a constantly accelerated clock-body:

$$t = \frac{c}{a}\sinh\frac{a}{c}\tau \qquad \text{t1,}$$

Time *t* is rest frame time, τ is the proper time, *a* is constant acceleration, *c* is the speed of light. The acceleration in t1 is that of an ideal rocket and this equation is sometimes used to introduce the topic of acceleration in Special and General Relativity and to present some of the novel implications and issues associated with accelerating an object for any substantial amount of time.[3] The acceleration is that of a point associated with the mass of the electron. Expressing t1 as proper time and resisting the temptation to simplify the equation, not setting *c* equal to unity (as is often done),

$$\tau = \frac{c}{a}\sinh^{-1}\frac{a}{c}t = (c/a)\ln\left[(c/a) + \sqrt{(a/c)^2 + 1}\right] \qquad \text{t2.}$$

[1] Some applications are presented here and some are suggested. More will be given in refinements to this introduction in consecutive submissions to this and appropriate journals.

[2] $G = 6.67408 \times 10^{-11} m^3 kg^{-1} s^{-2}$, all values: http://physics.nist.gov/cuu/Constants/index.html

[3] Marder, L. (1971) *Time and the Space-Traveller*. U of Pennsylvania P: Philadelphia.

The following expressions of the rest masses of the proton and neutron were found by considering the nature of the interaction of a particle and its antiparticle, specifically the electron and positron.

The reciprocal of the Compton frequency[4] ω_C and its extreme dilation under a nearly constant high acceleration were considered and compared to the interaction of an electron and proton at the Bohr radius. Hypothetical gravitational interaction between the particles at the Planck length[5] ℓ_P was considered; however, details of the heuristic and numerical exploration that led to the expressions below are omitted. The first meaningful expressions found in this process were that of the rest masses of the neutron and proton in terms of a hyper-accelerated point, accelerated for the short period equal to the reciprocal of Compton frequency.

The reciprocal of the Compton frequency τ_e and the reciprocal of any Compton-like frequency T_x defined similarly[6] by its rest mass m_x:

$$\omega_C^{-1} = \left(\frac{\mathrm{m_e} c^2}{\hbar}\right)^{-1} = \tau_e \qquad \text{CT1;}$$

$$\omega_x^{-1} = \left(\frac{\mathrm{m_x} c^2}{\hbar}\right)^{-1} = T_x \qquad \text{CT2.}$$

Thus, the rest mass of any elementary particle, composite particle, atom or any mass can be expressed by the rearrangement CT2:

$$m_x = \frac{\hbar \omega_x}{c^2} = \frac{\hbar}{T_x c^2} \qquad \text{MT.}$$

<u>Neutron and Proton Hypothetical Rest Mass</u>

Defining the rest mass of a particle this way (CT1, CT2, and MT) allowed for heuristic exploration and numerical experimentation on the nature of a particle-matter wave's phase and group velocities; the particle undergoing constant acceleration as would be described by t1 and t2. This led to the following hypothetical expressions of the rest masses of neutron and proton:

$$m_n = \frac{m_e}{2\pi\alpha} \sinh^{-1}\left[\left(\frac{4}{\pi}\right)\left(\frac{\alpha^4 \hbar c}{G m_e^2}\right)\right] = 1.675 \times 10^{-27}\,\mathrm{kg} = 939.617\,\mathrm{MeV}/c^2 \qquad \text{n1;}$$

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[4] $\omega_C = m_e c^2/\hbar$, where $m_e c^2$ is the rest energy of the electron and $\hbar$ is the reduced Planck constant. This the frequency of a photon created in annihilation of the electron's rest mass.

[5] $\ell_P = \sqrt{G\hbar/c^3} = 1.61620 \times 10^{-35}\,\mathrm{m}$, where G is the universal gravitational constant.

[6] Simply the frequency of one of the photons created in pair annihilation if one of the pair were at rest.

$$m_p = \frac{m_e}{2\pi\alpha}\sinh^{-1}\left[\left(\frac{4}{\pi}\right)^{1/2}\left(\frac{\alpha^4\hbar c}{Gm_e^2}\right)\right] = 1.673\times10^{-27}\,\text{kg} = 938.272\,\text{MeV}/c^2 \qquad \text{p1.}$$

Several questions immediately come to mind and only a few are addressed: Is this a novel coincidence specific to the neutron and proton; as digits are added to the gravitational constant, will they coincide with n1 and p1; if these expressions are relevant to the understanding of the relationship between gravitation and particle properties, can a general form of n1 and p1 be found and perhaps define the gravitational constant; can other well-established measures of fundamental masses be expressed similarly?

<u>Finding a General Hypothetical Rest Mass Expression</u>

Returning to the expression for the neutron and comparing it to that of the proton, a general expression might take the form:

$$m_x = \frac{m_e}{2\pi\alpha^{N/2}}\sinh^{-1}\left[\chi\left(\frac{\alpha^{n/2}\hbar c}{Gm_e^2}\right)\right] \qquad \text{G1.}$$

Hypothesizing that any mass can be expressed with G-1, with exponents of α, the fine structure constant, taking on values of integers and half integers:

$$N \text{ and } n = ..., -3, -2, -1, 0, 1, 2, 3, ...;$$

Chi χ is a real number[7], between zero and 2, specific to a particle and its properties (spin, gyromagnetic ratio, charge etc.). Note that because the fine structure constant contains the unit charge e, N and n must be restricted to integers (and half integers).

The ratio $\hbar c/\left(Gm_e^2\right)$ will be given the symbol R occasionally to simplify some calculations

$$R \equiv \frac{\hbar c}{Gm_e^2} = 5.70857\times10^{44} \qquad \text{R.}$$

Before presenting a short list of topical particle masses derived from G1, its relationship to t2 is demonstrated, the proper time of a point-like object undergoing constant acceleration.

Comparing t2 to G1:

$$\tau = \frac{c}{a}\sinh^{-1}\frac{a}{c}t$$

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[7] χ equals $4/\pi$ for the neutron and $2/\sqrt{\pi}$ for the proton. Note that $2\left(2/\pi\right)^2 = 2\left(4/\pi\right) = 8/\pi$.

and $$m_x = \frac{m_e}{2\pi\alpha^{N/2}} \sinh^{-1}\left[\chi\left(\frac{\alpha^{n/2}\hbar c}{Gm_e^2}\right)\right].$$

Where is time and acceleration in G1? Expressing the rest mass of electron with the Compton frequency (CT1), and rest mass of m_x with CT2 and substituting these into G1 and by correspondence with t2, finding a meaningful acceleration:

$$\hbar/\left(T_x c^2\right) = \frac{\hbar/\left(\tau_e c^2\right)}{2\pi\alpha^{N/2}} \sinh^{-1}\left(\chi\alpha^{n/2}R^{-1}\right),$$

Finding an acceleration a_x,

$$\frac{T_x}{2\pi\alpha^{N/2}} = \frac{c}{a} \rightarrow a_x = \frac{2\pi\alpha^{N/2}c}{T_x} = 2\pi\alpha^{N/2}c\omega_x,$$

$a_x = 2\pi\alpha^{N/2}c\omega_x$, comparing it to t2 and the hyperbolic arcsine's argument in G1,

$$\frac{aT_e}{c} = \chi\left[\frac{\alpha^{n/2}\hbar c}{Gm_e^2}\right] = \left(2\pi\alpha^{N/2}c\omega_x\right)\left(\frac{1}{c}\right)T_e,$$

where T_e is a very long rest frame time for the electron. The value of T_e will be shown in future letters to be of particular relevance to cosmology.[8] For now, we return to finding an expression that demonstrates the origins of G1 in t2.

Placing the results back into G1 we now have an expression that looks like t2,

$$\tau_e = \left(\frac{T_x}{2\pi\alpha^{N/2}}\right) \sinh^{-1}\left(2\pi\alpha^{N/2}\omega_x T_e\right) \qquad \text{TX1.}$$

Expressing $R^{-1} = \hbar c/\left(Gm_e^2\right)$ in terms of the Plank Length ℓ_P, the Compton Frequency ω_C, and the speed of light allows us to see a most important interpretation of TX1:

$$\ell_P = \sqrt{\frac{G\hbar}{c^3}};$$

[8] Note $T_e = 2\frac{\alpha^3\hbar^2}{Gm_e^2 m_n c} = \frac{3.10809\times10^{14}s}{\alpha^{1.5}} = 4.98592\times10^{17}s$

$$R^{-1} = \frac{\hbar c}{Gm_e^2} = \left(\frac{\hbar c}{G}\right)\left(\frac{c^2}{\hbar \omega_C}\right)^2 = \left(\frac{c^3}{G\hbar}\right)\left(\frac{c^2}{\hbar \omega_C}\right) = \frac{c^2}{\omega_C^2 \ell_P^2} = 5.70857 \times 10^{44} \qquad \text{R1.}$$

Substituting R1 into TX1, we now have G1 in a form that allows us to show the nature of a particle's mass in properties as that of an electromagnet-gravitational wave,

$$\tau_e = T_x \left(\frac{1}{2\pi\alpha^{N/2}}\right) \sinh^{-1}\left(\chi \frac{c^2}{\omega_C^2 \ell_P^2}\right) \qquad \text{G2.}$$

Recognizing that $c^2 / \left(\omega_C^2 \ell_P^2\right)$ can be interpreted as the ratio of superluminal phase velocity v_p and a very low group velocity v_g :

$$\frac{v_p}{v_g} = \frac{c^2}{\omega_C^2 \ell_P^2} = \left(\frac{c^2}{\omega_C \ell_P}\right)\left(\frac{1}{\omega_C \ell_P}\right), \text{ where } v_p = \frac{c^2}{\omega_C \ell_P} \text{ and } v_g = \omega_C \ell_P$$

The Compton Frequency,

$$\omega_C = \frac{m_e c^2}{\hbar} = \frac{\left(9.10938 \times 10^{-31}\right)\left(8.98755 \times 10^{16}\right)}{1.05457 \times 10^{-34}} = 7.76344 \times 10^{20} \,\text{s}^{-1}.$$

The Phase Velocity,

$$v_p = \frac{c^2}{\omega_C \ell_P} = \left(\frac{\hbar}{m_e c^2}\right)\frac{c^2}{\ell_P} = \frac{\hbar}{m_e \ell_P} = \frac{1.05457 \times 10^{-34}}{\left(9.10938 \times 10^{-31}\right)\left(1.61620 \times 10^{-35}\right)} \,\text{m/s} = 7.162952 \times 10^{30} \,\text{m/s}.$$

The Group Velocity,

$$v_g = \omega_C \ell_P = \left(\frac{m_e c^2}{\hbar}\right)\ell_P = \left(7.76344 \times 10^{20} \,\text{s}^{-1}\right)\left(1.61620 \times 10^{-35}\right) = 1.254727 \times 10^{-14} \,\text{m/s}.$$

So far the nature of the point that's undergoing acceleration MT, has not been addressed: $m_x = \frac{\hbar \omega_x}{c^2} = \frac{\hbar}{T_x c^2}$ allows us to find a value for the acceleration $2\pi\alpha^{N/2} c \omega_x$; if m_x equals the mass of the neutron as in n1, then

$$a_n = 2\pi\alpha c \omega_n = \left(2\pi\alpha c\right)\left(\frac{m_n c^2}{\hbar}\right) = 2\pi \frac{\alpha m_n c^3}{\hbar} = 1.96212 \times 10^{31} \, m/s^2.$$

This acceleration times the reciprocal of the Compton frequency divided by the speed of light as in t1:

$$\sinh\left(\frac{a_n\tau_e}{c}\right)=\sinh\left(2\pi\frac{\alpha m_n c^3}{\hbar c}\right)\left(\frac{\hbar}{m_e c^2}\right)=\sinh\left(2\pi\alpha\frac{m_n}{m_e}\right)=\left(\alpha^4\frac{4}{\pi}\right)\frac{\hbar c}{Gm_e^2}$$

Again, $(4/\pi)\alpha^{n/2}\hbar c\big/\left(Gm_e^2\right)=\chi\alpha^{n/2}R^{-1}$ is associated specifically with the neutron, and it is conjectured that any particle can be expressed similarly.
Rearranging to find meaning,

$$Gm_e^2\sinh\left(2\pi\alpha\frac{m_n}{m_e}\right)=\frac{2\alpha^4 hc}{\pi^2}=\left(\frac{2\alpha^3 hc}{\pi^2}\right)\frac{e^2}{4\pi\varepsilon_0\hbar c}$$

and

$$Gm_e^2\left[\frac{\pi^2}{2\alpha^3}\sinh\left(2\pi\alpha\frac{m_n}{m_e}\right)\right]=\frac{e^2}{4\pi\varepsilon_0};$$

We see something that looks like gravitation between electron and positron masses on the left and something looks like the electrostatic force between them the right. On the left, the field surrounding the electron has been hyper-inflated by $\left[\pi^2 m_e\big/\left(2\alpha^3\right)\right]\sinh\left[2\pi\alpha\left(m_n/m_e\right)\right]$.

Hypothetical Up and Down Quark Mass

Returning to the expression of the neutron's rest mass n1,

$$m_n=\frac{m_e}{2\pi\alpha}\sinh^{-1}\left(\frac{2\alpha^4 hc}{\pi^2 Gm_e^2}\right)=\frac{m_e}{2\pi\alpha}\sinh^{-1}\left(\frac{4}{\pi}\frac{\alpha^4\hbar c}{Gm_e^2}\right)\approx\frac{m_e}{2\pi\alpha}\ln\left(8\frac{\alpha^4}{\pi R}\right);$$

Using the natural log estimate for large arguments of hyperbolic arcsine, the up and down quark rest masses can be isolated from the neutron and proton's "quark soup" so as demonstrate G1's usefulness in defining particle properties and the hypothetical string-like structures[9] that underlie those properties.

$$m_n\approx\frac{m_e}{2\pi\alpha}\ln\left(8\frac{\alpha^4}{\pi R}\right)=\frac{m_e}{2\pi\alpha}\left[\ln\left(\frac{4}{\pi}\right)+\ln\left(2\frac{\alpha^4}{R}\right)\right]=\frac{m_e}{2\pi\alpha}\ln\left(\frac{4}{\pi}\right)+\frac{m_e}{2\pi\alpha}\ln\left(2\frac{\alpha^4}{R}\right);$$

$$m_n\approx m_u+\frac{m_e}{2\pi\alpha}\ln\left(2\frac{\alpha^4}{R}\right)=2m_d+\frac{m_e}{2\pi\alpha}\ln\left(\frac{\alpha^4}{R}\right).$$

[9] This will be the thesis of an effort more rigorous than this introduction.

Hypothetical Masses of Up $\left(m_u\right)$ and Down $\left(m_d\right)$ Quarks:

$$m_u = \frac{m_e}{2\pi\alpha}\ln\left(4/\pi\right) = 2.69220\, MeV/c^2;$$

$$m_d = \frac{m_e}{2\pi\alpha}\ln\left(\frac{8}{\pi}\right)^{\frac{1}{2}} = 5.20861 MeV/c^2.$$

These values fall comfortably with the current uncertainties. They are presented here to six digits so that as more precise measurements of the up and down quark masses, correspondence or non-correspondence with these hypothetical values might guide interpretation of G1 or confirm its relevance. Note also that a quick inspection p1 (the proton expression) will yield similar hypothetical masses for the proton's two up and one down quarks. Consider the following short list of topical particle rest mass expressions presented here as an introduction to G1 and the field component equations. G1 and the field component equations (in the conclusion) introduce an interdependent relationship between gravitational and electromagnet fields and their source interactions.

Z^0 Boson mass, m_{Z^0}

$$m_{Z^0} = \frac{m_e}{2\pi\alpha^2}\sinh^{-1}\left[\left(\frac{4}{\pi}\alpha^9\right)\left(\frac{\hbar c}{G m_e^2}\right)\right] = 1.6256\times10^{-25}\,\text{kg} = 91.19\,\text{GeV}/c^2$$

W Boson mass, $m_{w^\pm}$

$$m_{W^\pm} = \frac{m_e}{2\pi\alpha^2}\sinh^{-1}\left[\left(\alpha^{10.5}\sqrt{3}\right)\left(\frac{\hbar c}{G m_e^2}\right)\right] = 1.43304\times10^{-25}\,\text{kg} = 80.39\,\text{GeV}/c^2$$

Higgs Boson mass, m_{H^0}

$$m_{H^0} = \frac{m_e}{2\pi\alpha^2}\sinh^{-1}\left[\left(\frac{1}{8}\right)\left(\frac{\alpha^4\hbar c}{G m_e^2}\right)\right] = 2.23219\times10^{-25}\,\text{kg} = 125.217\ \text{GeV/c}^2.$$

We conjecture that all elementary, composite particle and isotope masses can be expressed with G1 (minimally six digits[10]) and with non-arbitrary values for N, n, and χ and that values for N, n, and χ are specific to a particle and directly associated with a particle's mass, charge, and spin. Returning to the suggestion in G1 that even rest mass can have a wave nature with very low group velocity and an extremely large phase velocity. The discovery of an all-pervasive dark matter suggests the nature of the dispersive medium if G1 is meaningful expression. Conjecture: there is an inverse relationship between a changing electric field and a gravitational field and this

[10] It can be shown that for masses substantially less than that of the electron's rest mass, the argument of the hyperbolic sine is small, and that the relationship between a particle's rest mass and χ/R is linear. Note the hypothetical neutrino flavor summation,

$$\sum m_\nu = \frac{m_e}{2\pi\alpha}\sinh^{-1}\left(\alpha^{23.5}/R\right) = 0.282587\text{eV/c}^2.$$

can be modeled as that of a wave with amplitude E_0 / ω_C , where E_0 is the electric field and ω_C is the Compton frequency; this wave moves along a chosen axis, x with phase angle $\pm\pi/2$ as follows:

$$\mathrm{G}(x,t) = -\frac{1}{\omega_C}\frac{\partial}{\partial t}\mathrm{E}(x,t) = -\frac{1}{\omega_C}\frac{\partial}{\partial t}E_0\cos\left(\frac{x}{\alpha^{n/2}\chi R\ell} - \omega t \pm \frac{\pi}{2}\right) \qquad \text{GE}$$

All constants are the same as defined above. We will show in subsequent letters that the frequency ω of GE can be associated with both photons and particles with non-zero rest mass, and that this frequency changes with the particle's acceleration as expected. The direction of this gravitational field in this wave is also conjectured as antiparallel to the changing magnetic field in an electro-magnetic wave.

An interpretation of the wave number $1/\left(\alpha^{n/2}\chi R\ell\right)$ is suggested below but not rigorously presented in this introduction.

A general expression of the relationship between the group and phase velocity can be written as

$$v_g \sinh\left(\frac{2\pi\alpha^{N/2}\tau_e}{T_x}\right) = \chi\alpha^{n/2}v_p \qquad \text{V1}$$

Or in terms of the rest mass of an the particle,

$$v_g \sinh\left(2\pi\alpha^{N/2}\, m_x/m_e\right) = \chi\alpha^{n/2}v_p \qquad \text{V2}$$

Taking the partial derivative of GE,

$$\mathrm{G}(x,t) = -\frac{E_0}{\omega_C}\left[\left(\frac{\partial\ell/\partial t}{\alpha^{n/2}\chi R\ell^2}x - \frac{\partial x/\partial t}{\alpha^{n/2}\chi R\ell}\right) + \omega + \frac{\partial\omega}{\partial t}t\right]\left\{\sin\left(\frac{x}{\alpha^{n/2}\chi R\ell} - \frac{\omega}{R}t + \pm\frac{\pi}{2}\right)\right\} \qquad \text{GE1}$$

Analyzing GE1, if $\partial x/\partial t = c$ is recognized as the speed of light, and ℓ given the Planck length then

$$\frac{1}{\omega_C}\frac{c}{\alpha^{n/2}\chi R\ell_P} = \frac{1}{\omega_C}\frac{Gm_e^2 c}{\alpha^{n/2}\chi\ell_P\hbar c} = \frac{\omega_C\ell_P}{\alpha^{n/2}\chi c^2} = \frac{c}{\alpha^{n/2}\chi\omega_C\ell_P R} = \frac{\omega_C\ell_P}{\alpha^{n/2}\chi c^2},$$

And we find the superluminal phase velocity in V1 and V2,

$$v_p = \frac{c^2}{\omega_C \ell_P} = 7\times 10^{30}\,\text{m/s.}$$

Although the total energy of an *electro-magnetic* wave moves with the group velocity, perhaps it is a small part of that total energy, the gravitational part, moving at this extremely high speed, a speed not restricted by Maxwell's Equations for *electro-magnetic* waves, which is responsible for the spin interactions that appear to occur simultaneously in the lab frame?

The relationship between the half-integers n and N, and the real number χ (in G1, GE, and GE1) and a particle's charge and intrinsic spin, will be shown in the rigorous development of this model. Demonstrating that GE1 represents both the electro-magnetic and gravitational interactions between the electron and positron and between any two particles, charge or uncharged, is reserved for later submissions to PR and can be considered an extension of this short introduction. Rigorous development of this model will necessarily include a generalization to 4D, will include the complex plane, and most importantly, experimental references and measure[11]. This is necessary because the dispersive nature of the waves in GE1 and the suggestion that the dispersive medium is that of dark matter.

[11] Recent experimentally derived values for several particle masses drove and will continue to drive the development of this model and theory. The author recognizes all sources of these values as principal in this model's origins and development.